The Born Rule from Geometry?

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This is a (perhaps misguided) attempt to derive the Born Rule from geometry. I'm not sure it's needed as Gleason's derivation [a] from unitarity seems convincing to me – though not to everybody.

Also, this may just be a derivation from unitarity underneath. I'm not sure.

Consider a spherical wave such as arises from scattering. The S-wave (angular momentum 0) wave function is

$$\psi = A_S \frac{e^{ikr}}{r}$$

The probability density for observing the particle at radius r is

$$\rho = |\psi|^2 = \frac{|A_S|^2}{r^2}$$

The probability of observing the particle on a surface at radius r in angular internal $d\Omega$ is

$$P = |\psi|^2 r^2 d\Omega = \frac{|A_S|^2}{r^2} \times r^2 d\Omega = |\mathrm{A_S}|^2 d\Omega$$

via the Born rule. It is independent r.

If we use a different rule, say $\rho = |\psi|^n$, that won't work any n other than two (well and zero). The probability will be

$$P = \frac{|A_S|^{2n}}{r^{2n}} \times r^2 d\Omega = \frac{|A_S|^{2n}}{r^{2n-2}} d\Omega$$

which gives a probability into $d\Omega$ that is falling. Unitarity is violated. It would appear that the Born rule is the only possible rule.

I think this is only a concrete example of Gleason's theorem in action.

[a] https://en.wikipedia.org/wiki/Gleason%27s_theorem#:~:text=for%20dimension%202.-,Deriving%20the%20state%20space%20and%20the%20Born%20rule,must%20add%20up%20to%201.